EE3124 Tutorial 3 (Solution)
AC Machine Fundamentals

Name: **Student No.:**

Q1 - Why does switching the current flows in any two phases reverse the direction of rotation of a three-phase AC motor?

Solution

Switched two phases of current then gives the net magnetic flux density in the stator:

$$B_{net} = B_M \sin\omega t \angle 0^o + B_M \sin(\omega t - 240^o)\angle 120^o + B_M \sin(\omega t - 120^o)\angle 240^o$$

Each of the three component magnetic fields can now be broken down into its x and y components:

$$B_{net}(t) = B_M \sin\omega t\,\hat{x} - [0.5 B_M \sin(\omega t - 240^o)]\hat{x} + \left[\frac{\sqrt{3}}{2} B_M \sin(\omega t - 240^o)\right]\hat{y}$$

$$- [0.5 B_M \sin(\omega t - 120^o)]\hat{x} - \left[\frac{\sqrt{3}}{2} B_M \sin(\omega t - 120^o)\right]\hat{y}$$

$$B_{net}(t) = [B_M \sin\omega t - 0.5 B_M \sin(\omega t - 240^o) - 0.5 B_M \sin(\omega t - 120^o)]\hat{x}$$

$$+ \left[\frac{\sqrt{3}}{2} B_M \sin(\omega t - 240^o) - \frac{\sqrt{3}}{2} B_M \sin(\omega t - 120^o)\right]\hat{y}$$

Using the angle-addition trigonometric identities,

$$B_{net}(t) = \left[B_M \sin\omega t + \frac{1}{4} B_M \sin\omega t\right.$$

$$\left. - \frac{\sqrt{3}}{4} B_M \cos\omega t + \frac{1}{4} B_M \sin\omega t + \frac{\sqrt{3}}{4} B_M \cos\omega t \right]\hat{x}$$

$$+ \left[-\frac{\sqrt{3}}{4} B_M \sin\omega t + \frac{3}{4} B_M \cos\omega t + \frac{\sqrt{3}}{4} B_M \sin\omega t + \frac{3}{4} B_M \cos\omega t\right]\hat{y}$$

$$B_{net}(t) = [1.5 B_M \sin\omega t]\hat{x} + [1.5 B_M \cos\omega t]\hat{y}$$

The magnetic field rotates in a clock-wise direction.

Note:

$$B_{net} = B_M \sin\omega t \angle 0^o + B_M \sin(\omega t - 120)\angle 120^o + B_M \sin(\omega t - 1240^o)\angle 240^o$$

$$B_{net}(t) = [1.5 B_M \sin\omega t]\hat{x} - [1.5 B_M \cos\omega t]\hat{y}$$

Q2 - A three-phase Δ- connected synchronous generator is rotating at 1800 rpm. The rotor peak magnetic field is 0.3 T. The stator diameter of the generator is 0.3 m and the coil length is 0.3 m. Each coil has 30 turns. What is the rms terminal voltage of this generator?

Solution

Flux in the generator: $\phi = \text{area} \cdot B = 0.3 \times 0.3 \times 0.3 = 0.027\text{Wb}$

Velocity of the rotor: $\omega = 2\pi f = 2\pi\frac{1800}{60} = 188.4 \text{ rad/s}$

The peak value of the induced voltage: $\hat{E} = N\phi\omega = 30 \times 0.027 \times 188.4 = 152.6\text{V}$

The generator is Δ - connected. phase voltage = line voltage = $\frac{152.6}{\sqrt{2}} = 107.9\text{V}$

Q3 - Use the correct word from the word bank to complete the sentence.

increases	four	amplifies	rotor	output voltage	two	output current
torques	attenuates	equal to	two-pole	inductance	four	times
stator	vibration	resonance	four-pole	reduces	twice	

i) Stator windings fractional-pitch ___________ the magnitude of the output voltage slightly, but ___________ the harmonic components of voltage drastically, resulting in a much smoother ___________ from the machine.

ii) In a ___________ pole machine, the mechanical speed of rotation of the magnetic field in revolutions per second is ___________ the electric frequency in hertz.

iii) A generator with ___________ winding, the electrical frequency of the current is ___________ the mechanical frequency of rotation.

iv) AC machines differ from dc machines in that their armature windings are almost always located on the ___________, while their field windings are located on the ___________.

v) The distributed windings induce harmonics in the generated voltage of ac machine. The interaction of stator and rotor slot harmonics produces parasitic ___________. Thus, they introduce ___________ and noise in the machine.

Solution

i) Stator windings fractional-pitch _reduces_ the magnitude of the output voltage slightly, but _attenuates_ the harmonic components of voltage drastically, resulting in a much smoother _output voltage_ from the machine.

ii) In a _two_ pole machine, the mechanical speed of rotation of the magnetic field in revolutions per second is _equal to_ the electric frequency in hertz.

iii) A generator with _four-pole_ winding, the electrical frequency of the current is _twice_ the mechanical frequency of rotation.
iv) AC machines differ from dc machines in that their armature windings are almost always located on the _stator_, while their field windings are located on the _rotor_.
v) The distributed windings induce harmonics in the generated voltage of ac machine. The interaction of stator and rotor slot harmonics produces parasitic _torques_. Thus, they introduce _vibration_ and noise in the machine.

Q4 - The first ac power system in the USA ran at a frequency of 133 Hz. If the ac power for this system were produced by a 4-pole generator, how fast would the shaft of the generator have to rotate?

Solution

The equation for the speed of the shaft is

$$n_{sm} = \frac{120 f_{se}}{P} = \frac{120(133 \text{ Hz})}{4} = 3990 \text{ r/min}$$

Q5 - Develop a table showing the speed of magnetic field rotation in ac machines of 2, 4, 6, 8, 10, 12, and 14 poles operating at frequencies of 50, 60, and 400 Hz.

Solution

The equation relating the speed of magnetic field rotation to the number of poles and electrical frequency is

$$n_{sm} = \frac{120 f_{se}}{P}$$

The resulting table is

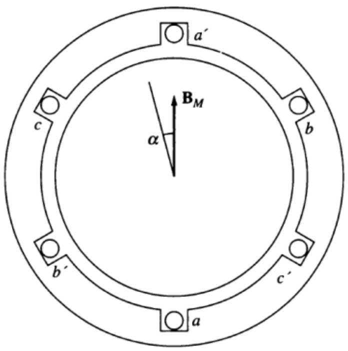
Number of Poles	f_e = 50 Hz	f_e = 60 Hz	f_e = 400 Hz
2	3000 r/min	3600 r/min	24000 r/min
4	1500 r/min	1800 r/min	12000 r/min
6	1000 r/min	1200 r/min	8000 r/min
8	750 r/min	900 r/min	6000 r/min
10	600 r/min	720 r/min	4800 r/min
12	500 r/min	600 r/min	4000 r/min
14	428.6 r/min	514.3 r/min	3429 r/min

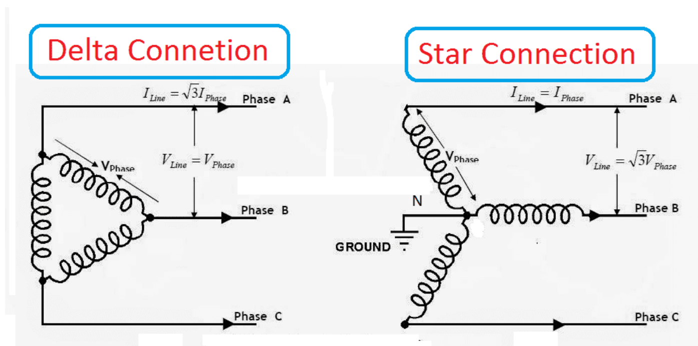
Q6 - The following information is known about the simple two-pole generator in following figure. The peak flux density of the rotor magnetic field is 0.2 T, and the mechanical rate of rotation of the shaft is 3600 r/min. The stator diameter of the machine is 0.5 m, its coil length is 0.3 m, and there are 15 turns per coil. The machine is Y-connected.

(a) What are the three phase voltages of the generator as a function of time?

(b) What is the rms phase voltage of this generator?

(c) What is the rms terminal voltage of this generator?





Solution

The flux in this machine is given by

$$\phi = 2rlB = dlB$$

where *d* is the diameter and *l* is the length of the coil. Therefore, the flux in the machine is given by

$$\phi = (0.5 \text{ m})(0.3 \text{ m})(0.2 \text{ T}) = 0.03 \text{ Wb}$$

The speed of the rotor is given by

$$\omega = (3600 \text{ r/min})(2\pi \text{ rad})(1 \text{ min}/60 \text{ s}) = 377 \text{ rad/s}$$

(a) The magnitudes of the peak phase voltages are thus

$$E_{max} = N_C\phi\omega$$
$$= (15 \text{ turns})(0.03 \text{ Wb})(377 \text{ rad/s}) = 169.7 \text{ V}$$

and the three phase voltages are

$$e_{aa'}(t) = 169.7 \sin 377t \quad V$$
$$e_{bb'}(t) = 169.7 \sin (377t - 120°) \quad \text{V}$$
$$e_{cc'}(t) = 169.7 \sin (377t - 240°) \quad \text{V}$$

(b) The rms phase voltage of this generator is

$$E_A = \frac{E_{max}}{\sqrt{2}} = \frac{169.7 \text{ V}}{\sqrt{2}} = 120 \text{ V}$$

(c) Since the generator is Y-connected,

$$V_T = \sqrt{3}E_A = \sqrt{3}(120 \text{ V}) = 208 \text{ V}$$

Q7 - A three-phase Y-connected four-pole winding is installed in 24 slots on a stator. There are 40 turns of wire in each slot of the windings. All coils in each phase are connected in series. The flux per pole in the machine is 0.060 Wb, and the speed of rotation of the magnetic field is 1800 r/min.
(a) What is the frequency of the voltage produced in this winding?
(b) What are the resulting phase and terminal voltages of this stator?

Solution

(a) The frequency of the voltage produced in this winding is

$$f_{se} = \frac{n_{sm}P}{120} = \frac{(1800 \text{ r/min})(4 \text{ poles})}{120} = 60 \text{ Hz}$$

(b) There are 24 slots on this stator, with 40 turns of wire per slot. Since this is a four-pole machine, there are two sets of coils (in 8 slots) associated with each phase. The voltage in the coils in *one pair* of slots is

$$E_A = \sqrt{2}\pi N_C \phi f = \sqrt{2}\pi (40 \text{ t})(0.060 \text{ Wb})(60 \text{ Hz}) = 640 \text{ V}$$

There are eight slots associated with each phase, and the all of the coils in a slot are connected in series, so the total phase voltage is

$$V_\phi = 4(640 \text{ V}) = 2560 \text{ V}$$

Since the machine is Y-connected, $V_L = \sqrt{3}V_\phi =$ 4434 V